

XVI. Proposal of a Method for measuring Degrees of Longitude upon Parallels of the Æquator, by J. Michell, B. D. F. R. S.

Read May 8, 1766. **T**HERE have been already several attempts made towards discovering the figure of the earth, by measuring the length of a degree of the meridian in different latitudes: now if these measures had been sufficiently accurate and numerous, and we could also depend upon the uniformity of the earth's surface, we might then immediately discover from them the form sought; but these measures, not agreeing exactly to any certain rule, leave us still in some degree at a loss. It is therefore much to be wished, that more measurements of degrees upon the meridian were to be made, in order to determine with greater accuracy a question of this importance.

But what would tend yet more to determine this matter, would be the measurement of degrees of longitude, as well as those of latitude. Astronomers have indeed expressed their wishes that this might be done; and though no attempt has been hitherto made towards it, yet as it is probable, that such measurements may some time or other take place, it will not be amiss to suggest a method, which will admit of more exactness than any I have seen proposed for this purpose, all of which, depending upon an observation of the time, are therefore liable to an error of fifteen seconds of a degree for every second of time; but the method, I mean to recommend, stands upon the same foundation with the
 1 measure-

measurement of a degree of the meridian, and, the instruments being equally good, and the number of miles to be measured the same, the exactness of it, to that of a degree of the meridian, will be in the proportion of the sine of the latitude to the radius very nearly.

In TAB. V. Fig. 7. let AB represent the æquator; P the pole; DLEF a parallel of the æquator; PEC a meridian passing through the station E; PLMN a meridian passing through another station M; and let AMEB be a great circle cutting the meridian PEC at right angles in the point E.

Then in the spherical triangle AMN, right angled at N, we shall have $R : \text{Cof. AM} [\text{Sin. ME}] :: \text{Tan.}$

$\text{MAN} : \text{Co-Tan. AMN} :$ hence $\frac{\text{Tan. MAN}}{R} \times \text{Sin.}$

$\text{ME} = \text{Co-Tan. AMN} ;$ but $\text{Tan. MAN},$ being the Tangent of the latitude of the given place E, and

therefore given, the quantity $\frac{\text{Tan. MAN}}{R}$ will likewise

be given, and greater or less than unity in the proportion of the Tan. of the latitude to the R. The Co-Tan. therefore of the angle AMN, that is the Tan. of the complement of the angle AMN to 90° . will be greater or less than the Sine of the Arc ME, in the proportion of the Tan. of the latitude of the place, to the R. And consequently, whilst the Arc ME is small (in which case the Sine, Arc, and Tangent differ very little from each other) the angular deviation of the intersection of the meridian PLMN with the great circle AMEB, from a right angle, will contain more or fewer degrees, &c. than the Arc ME nearly in the (same) proportion of the tan. of the latitude of the place to the R.

By

By this means then, the latitude of the place and the angle PME (contained between the meridian PMN and the great circle AMB) being given, the length of the Arc ME will likewise be given, with great exactness. But I must observe, that as the angles PEM and PME must be taken by the observation of some star near the pole, they will be less accurate, when reduced to the plain of the horizon, than at the pole, in the proportion of the Sine of the distance between the pole and zenith, that is the Cof. of the latitude, to the R, which with the proportion just mentioned of the Tan. of the latitude to the R, makes the accuracy of this method upon the whole, when compared with that of the measurement of a degree of the meridian, in the proportion of the Tan. multiplied into the Cof. of the latitude, to the square of the R. very nearly; but the Tan. of any angle into its Cof. is equal to the Sin. into the R. whence this proportion is the same as the Sin. into the R. to the square of the R. and dividing both by the R. simply as the Sin. of the latitude, to the R. as above.

Having got the length of the arc ME, of a great circle, in degrees, &c. together with the distance of the two stations M and E, it is easy to conclude from thence the length of a degree of the parallel of latitude, at the place of observation, which will be the same, without sensible error, as it would be, supposing the earth was an exact sphere, to the same scale, with the degree of a great circle just found.

For in Fig. 8. let APB represent a section of the earth through its axis PCH; ACB an æquatorial diameter; AD the radius of curvature at the point A; and PH the radius of curvature at the

point P; DFH the evolute of the curve AEP; EF the radius of curvature at the point E; (which we will suppose to have the same latitude with the point E in Fig. 7.) and let EF be produced till it cuts the axis PH in G: then with the radius EG and centre G, describe the arc IEK, which will be the least circle, that can touch the curve AEP at the point E, without cutting it. Let now the curve PEA, the line EG, and the arc IEK revolve about PH as an axis, and, PE being equivalent to PE in the former figure, the point E in the latter figure will describe the parallel DEF in the former; AEP at the same time describing the surface of the earth, and IK describing a portion of a sphere, which will be every where a tangent to the parallel DEF, and whose centre will be G. The curvature therefore of this sphere will be less than the curvature of the earth, in the direction of the meridian, at the point E, as the radius GE is greater than the radius FE; but this, in moderate distances, can cause no sensible error. The difference between AD the radius of curvature, at the point A, on the earth's surface, and the line AC, according to that hypothesis, which makes it the greatest, does not exceed one sixtieth part of the whole, and upon the same hypothesis, the part FG of the line EG, supposing E to be in the latitude of 45° , would not exceed $\frac{1}{130}$ part of the whole. If then we take any other point upon the surface of the earth as M, at a small distance from E, the distance between that point and the sphere described by the arc IK, will be only $\frac{1}{130}$ part of the versed sine of the arc EM, and the perpendicular standing upon the surface of the earth at M, will be inclined to the perpendicular standing

standing upon the sphere in an angle, which is equal to $\frac{1}{130}$ part of the angle subtended by the arc EM. And in higher latitudes these quantities will be still less. Let us now return to Fig. VII. and supposing the point E to be situated in latitude 45° , let the arc EM cutting PE at right angles, consist of 2° (near 140 statute miles) then will the side PM, of the triangle PME, consist of $45^\circ 2' 5'' \frac{1}{2}$, and consequently, if LM in Fig. 7. be supposed to correspond to EM in Fig. 8. the distance of these two points E and M, in the latter Fig. will be only $2' 5'' \frac{1}{2}$, the $\frac{1}{130}$ part of the versed sine of which is a little more than $\frac{1}{3}$ of an inch, to the radius of the earth, which will therefore be the distance of the point M upon the earth's surface, and the point of the imaginary sphere, described by IK, immediately over it. Hence also, the inclination of the real perpendicular at M, and the imaginary one standing upon the arc IK, at the same place, to each other, will be something less than a second, a quantity in itself almost too small to be regarded, unless the instruments made use of are both very large and very excellent in their kinds, and which, being wholly in the plane of the meridian, will produce an error, that must be perfectly insensible, with any instruments whatsoever, in an observation of the angle PME, Fig. 7. which will therefore, to all intents and purposes, be the same, as if the curvature of the earth in the direction of the meridian, and in the direction of ME or LE were accurately the same.

I have supposed the arc ME, to stand at right angles to the meridian PE, which passes through one of the extrem stations; the method here proposed is,

however, liable to the least error, when the meridian cuts the arc to be measured at right angles in the middle of it; but this makes so very small a difference, that it is not worth regarding; nor is it indeed necessary, that the arc should not deviate two or three degrees from right angles with the meridian, at that end where it cuts it most nearly at right angles, in case the situation and circumstances of the country should make this more convenient, the errors, that would be occasioned by such a deviation, being too small to affect the conclusion. And if this deviation was still much greater, and the length of a degree of the meridian at the same place was known, it would be very easy to make the necessary corrections.

It will perhaps be objected, that the method above proposed depends, in some measure, upon time, as well as others, the finding of the meridian not being to be performed without it; but I must observe, that the motion of the pole star, by which I propose to find the meridian, being slower than that of a star at the æquator, nearly in the proportion of 30 to 1, this method will admit of an exactness greater in the same proportion (except the reduction of the Sin. to the R. before mentioned) than those observations, by which we endeavour to find the difference of the longitude of two places, by the difference of the time of the sun, or a star's coming to their respective meridians.

The method above proposed will likewise require different instruments from those commonly in use; but admitting, that instruments of equal radius are capable of equal exactness, this method will admit of the same exactness with the observations of a degree of the meridian, except the before-mentioned limitation.

tion. Nor would the instruments for this purpose, if well contrived, be either less portable, or more expensive, than those for measuring a degree on the meridian; the same telescope which would be necessary for finding the meridian, would serve likewise for tracing the arc of a great circle; but it is beside my present purpose to consider any farther what kind of instruments would be most proper upon this occasion, or what would be the best manner of constructing them.

I must not, however, dismiss the present subject, without observing, that, by means of the above-mentioned method, a country not too near the æquator, nor attended with any other unfavourable circumstances, might be laid down with wonderful exactness. By running a great circle nearly East and West through the midst of it, we should get the longitude of all the places, the great circle passed over; and if, by means of the meridian telescope, we should trace meridians through a few of these places, as far North and South, as the survey was intended to be carried, we should then have a number of stations, in several parts of the country, whose longitudes, with respect to one another, would be very accurately determined, and to which other places might easily be referred, when the length of a degree of longitude in those situations was known.

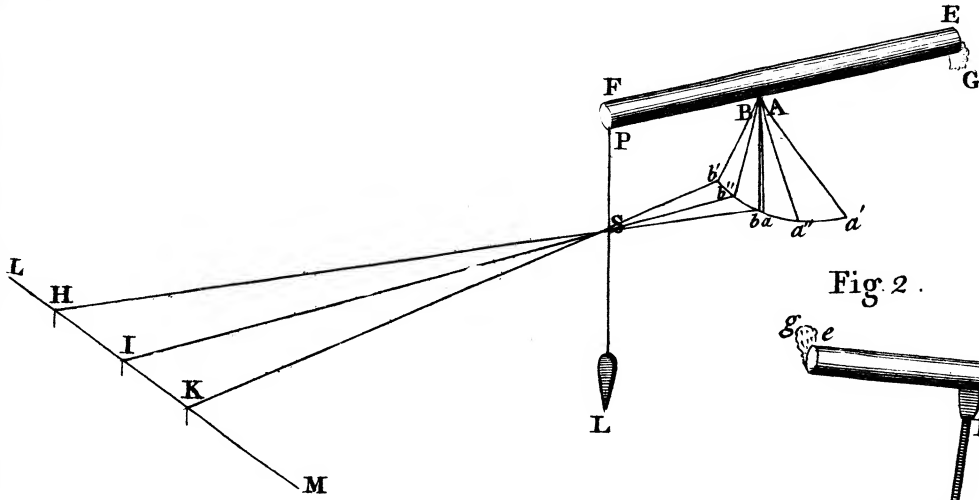


Fig. 2.

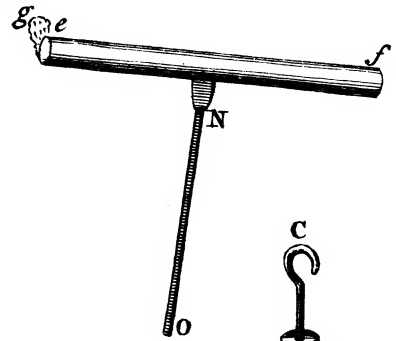


Fig. 4.



Fig. 5.



Fig. 6.



Fig. 3.

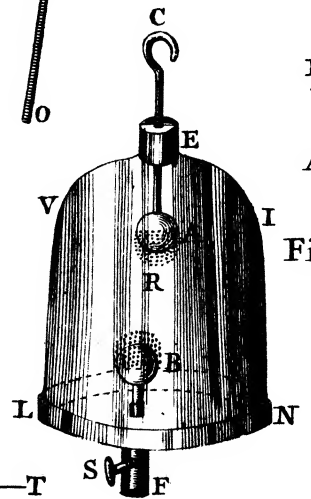
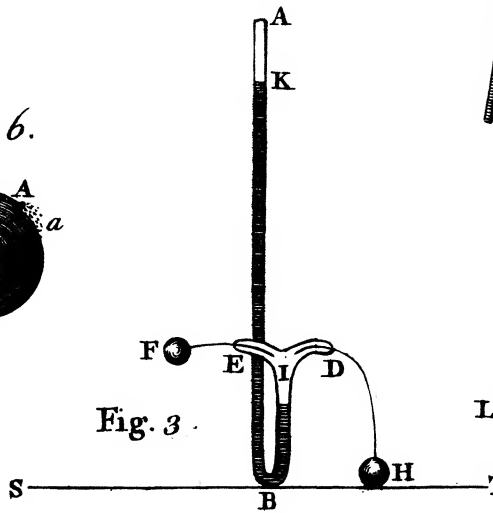


Fig. 1.

Fig. 7.

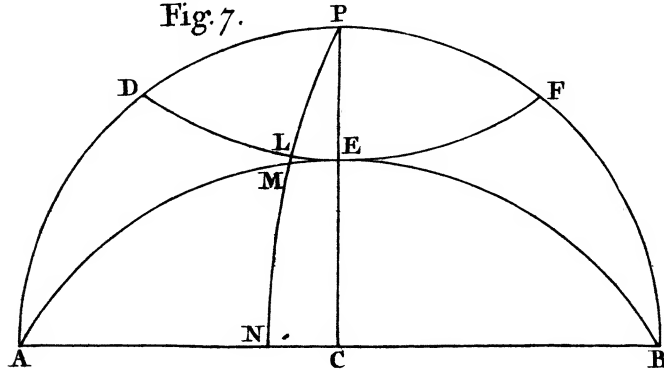


Fig. 8.

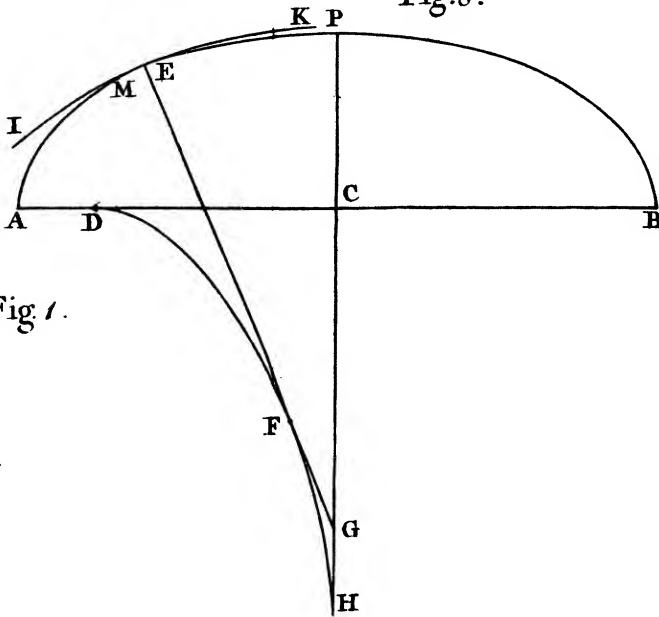


Fig. 1.

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